

A Level Statistics

AQA Past Exam Questions

TOPIC: Hypothesis Testing Paired t Test

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions **on paper**
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Unless otherwise stated, statistical tests should be carried out at the 5% significance level.
- When a calculator is used, the answer should be given to three significant figures unless otherwise stated.

Information

- **You may use the** booklet 'Statistical Formulae and Tables'
- There are **8** questions in this question paper. The total mark for this paper is **72**
- The marks for **each** question are shown in brackets – use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.
- Check your answers if you have time at the end.

Q3	Solution	Mark	Total	Comment																												
(a) (i)	H_0 pop mean diff $\mu_d = 0$ H_1 pop mean diff $\mu_d > 0$ [Before – After] 1 tail 5% $d = \text{Before} - \text{After}$	B1		Hypotheses consistent with d																												
	<table border="1"> <thead> <tr> <th></th> <th>A</th> <th>B</th> <th>C</th> <th>D</th> <th>E</th> <th>F</th> </tr> </thead> <tbody> <tr> <td>d</td> <td>0.8</td> <td>-0.3</td> <td>0.6</td> <td>1.9</td> <td>0.3</td> <td>2.6</td> </tr> <tr> <th></th> <th>G</th> <th>H</th> <th>I</th> <th>J</th> <td colspan="2"></td> </tr> <tr> <td>d</td> <td>0.9</td> <td>-0.9</td> <td>-0.7</td> <td>1.6</td> <td colspan="2"></td> </tr> </tbody> </table>		A	B	C	D	E	F	d	0.8	-0.3	0.6	1.9	0.3	2.6		G	H	I	J			d	0.9	-0.9	-0.7	1.6			M1 A1		Differences (ignore signs) At least 8 correct differences
		A	B	C	D	E	F																									
	d	0.8	-0.3	0.6	1.9	0.3	2.6																									
		G	H	I	J																											
	d	0.9	-0.9	-0.7	1.6																											
	$\bar{d} = 0.68$ $s = 1.135$ $n = 10$	m1		Attempt to evaluate \bar{d} and s Condone 1.0768 here																												
	$t = \frac{0.68 - 0}{\frac{1.135}{\sqrt{10}}} = 1.89$	m1 A1		Wholly correct method for t (condone small slip in \bar{d} , s) (\pm) 1.89 (1.85 – 1.95) or $p = 0.0453$																												
	$df = 9$ $cv = 1.833$ $1.833 < 1.89$ or $0.0453 < 0.05$	B1		for correct cv or p compared with 0.05																												
	Reject H_0	A1dep		correct conclusion dep ts/cv correct																												
Significant evidence to suggest that after the introduction of the <u>bonus/ (scheme)</u> there was a <u>reduction</u> in the <u>mean absenteeism</u> level.	E1dep		correct conclusion in context Not too definite																													
		9																														
(ii)	<u>Differences in absenteeism levels</u> are normally distributed	B1	1	Normally distributed with some context.																												

Q1	Solution	Marks	Total	Comments																								
(a)	H_0 pop mean diff $\mu_d = 0$	B1		Hypotheses																								
	H_1 pop mean diff $\mu_d > 0$ 1 tail 1%																											
	$d = \text{Robusta} - \text{Arabica}$																											
	<table border="1"> <thead> <tr> <th></th> <th>A</th> <th>B</th> <th>C</th> <th>D</th> <th>E</th> </tr> </thead> <tbody> <tr> <td>d</td> <td>133</td> <td>104</td> <td>-87</td> <td>-86</td> <td>142</td> </tr> <tr> <th></th> <th>F</th> <th>G</th> <th>H</th> <th>I</th> <th>J</th> </tr> <tr> <td>d</td> <td>101</td> <td>185</td> <td>167</td> <td>209</td> <td>97</td> </tr> </tbody> </table>		A	B	C	D	E	d	133	104	-87	-86	142		F	G	H	I	J	d	101	185	167	209	97	M1		Differences (consistent with H_1) At least 5 correct differences
		A	B	C	D	E																						
	d	133	104	-87	-86	142																						
		F	G	H	I	J																						
	d	101	185	167	209	97																						
	$\bar{d} = 96.5$ $s = 103.2$ $n = 10$	m1dep		Attempt to evaluate \bar{d} and s																								
	$t = \frac{96.5 - 0}{103.2 / \sqrt{10}} = 2.96$	B1		Use of $\frac{s}{\sqrt{n}}$ ft in ts $n = 9$ or 10																								
$df = 9$ $cv = 2.821$	m1		Method for t																									
$2.96 > 2.821$	A1		(\pm) 2.96 (2.85 – 3.05) or $p=0.008$																									
Reject H_0	B1		for correct cv or $p = 0.00803 < 0.01$ gains M1m1B1m1A1B1																									
Significant evidence to suggest Robusta produces a higher mean/average yield of green beans. Or Scientist's belief supported by given data	E1dep		correct conclusion in context 1 tail																									
			8																									

Q2	Solution	Marks	Total	Comments																												
(a)(i)	Assume that differences in pulse/rate measurements are normally distributed and that the 12 people can be regarded as a random sample.	E1 E1	2	Normal dist and random sample in context																												
(ii)	<p>H_0 pop mean diff/ $\mu_d = 0$ H_1 pop mean diff/ $\mu_d \neq 0$ 2 tail 5%</p> <p>$d = \text{After} - \text{before}$</p> <table border="1"> <thead> <tr> <th></th> <th>1</th> <th>2</th> <th>3</th> <th>4</th> <th>5</th> <th>6</th> </tr> </thead> <tbody> <tr> <td>d</td> <td>5</td> <td>6</td> <td>1</td> <td>0</td> <td>-1</td> <td>0</td> </tr> <tr> <th></th> <th>7</th> <th>8</th> <th>9</th> <th>10</th> <th>11</th> <th>12</th> </tr> <tr> <td>d</td> <td>5</td> <td>1</td> <td>-1</td> <td>-5</td> <td>4</td> <td>2</td> </tr> </tbody> </table> <p>$\bar{d} = 1.417$ $s = 3.175$ $n = 12$</p> $t = \frac{1.417 - 0}{\frac{3.175}{\sqrt{12}}} = 1.545$ <p>$df = 11$ $cv = \pm 2.201$ $1.545 < 2.201$</p> <p>Accept H_0</p> <p>No significant evidence to suggest that there is a difference in average pulse rates before and immediately after minor dental treatment</p>		1	2	3	4	5	6	d	5	6	1	0	-1	0		7	8	9	10	11	12	d	5	1	-1	-5	4	2	B1 M1 mldep diffs M1 A1 B1 A1 E1	8	Hypotheses sc 0's ignored B1 M1 m1 M1 only Differences – can be reverse sign attempt to find \bar{d} , s seen/labelled Use of $\frac{s}{\sqrt{12}}$ ft on 's' (\pm) 1.54 or 1.55 (1.53 – 1.56) (or $p = 0.15$ compared with 0.025) for correct cv Correct conclusion with consistent comparison Correct conclusion in context
	1	2	3	4	5	6																										
d	5	6	1	0	-1	0																										
	7	8	9	10	11	12																										
d	5	1	-1	-5	4	2																										
(iii)	Type II error	B1	1																													

Q2	Solution	Marks	Total	Comments																												
(a)	<p>H_0 pop mean diff or $\mu_d = 0$ H_1 pop mean diff or $\mu_d > 0$ [QC – DD] 1 tail 5% $d = QC - DD$</p>	B1		Hypotheses consistent with d																												
	<table border="1" data-bbox="217 443 746 577"> <thead> <tr> <th></th> <th>A</th> <th>B</th> <th>C</th> <th>D</th> <th>E</th> <th>F</th> </tr> </thead> <tbody> <tr> <td>d</td> <td>+5</td> <td>0</td> <td>+4</td> <td>+3</td> <td>-1</td> <td>-1</td> </tr> <tr> <th></th> <th>G</th> <th>H</th> <th>I</th> <th>J</th> <th>K</th> <th>L</th> </tr> <tr> <td>d</td> <td>+4</td> <td>+2</td> <td>+8</td> <td>+4</td> <td>+9</td> <td>+2</td> </tr> </tbody> </table>		A	B	C	D	E	F	d	+5	0	+4	+3	-1	-1		G	H	I	J	K	L	d	+4	+2	+8	+4	+9	+2	M1 A1		Differences (ignore signs) At least 8 correct differences PI
		A	B	C	D	E	F																									
	d	+5	0	+4	+3	-1	-1																									
		G	H	I	J	K	L																									
	d	+4	+2	+8	+4	+9	+2																									
	$\bar{d} = 3.25$ $s = 3.166$ $n = 12$	m1dep		Attempt to evaluate \bar{d} and s sc 0 excluded gives $\bar{d} = 3.545$ M1A1m1 max																												
	$t = \frac{3.25 - 0}{3.166 / \sqrt{12}} = 3.56$	B1 m1		Use of $\frac{s}{\sqrt{n}}$, $n = 12$ ft in ts oe PI Method for t																												
	$df = 11$ $cv = 1.796$ $1.796 < 3.56$	A1		(\pm) 3.56 (3.50 – 3.60)																												
	Reject H_0	B1		for correct cv Allow p value = 0.00225(for m1 B1 m1A1) compared with 0.05(B1)																												
Significant evidence to suggest that: the belief is correct OR	A1 dep		correct conclusion																													
the time taken, after placing on ward, before a box of <i>Chok Delights</i> is opened is, on average, less than that for a box of <i>Quality Chox</i>	E1dep	10	correct conclusion in context																													

Q	Solution	Marks	Total	Comments																																																					
1(a)	16 birds/feathers can be regarded as a random sample. Differences in yellowness index can be assumed to be normally distributed.	E1 E1	2	At least one in context for E2 sc B1 for both reasons, no context																																																					
1(b)	H_0 pop mean diff $\mu_d = 0$ H_1 pop mean diff $\mu_d > 0$ 1 tail 1% $d = \text{Typical} - \text{Odd}$	B1		Consistent with differences Odd – Typical $H_1 \mu_d < 0$																																																					
	<table border="1" style="width: 100%; text-align: center;"> <tr> <td></td> <td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td> </tr> <tr> <td>d</td> <td>+</td><td>-</td><td>+</td><td>-</td><td>-</td><td>+</td><td>+</td><td>+</td> </tr> <tr> <td></td> <td>.069</td><td>.028</td><td>.109</td><td>.041</td><td>.018</td><td>.014</td><td>.249</td><td>.08</td> </tr> <tr> <td></td> <td>9</td><td>10</td><td>11</td><td>12</td><td>13</td><td>14</td><td>15</td><td>16</td> </tr> <tr> <td>d</td> <td>+</td><td>+</td><td>+</td><td>+</td><td>+</td><td>+</td><td>+</td><td>+</td> </tr> <tr> <td></td> <td>.032</td><td>.189</td><td>.119</td><td>.37</td><td>.386</td><td>.241</td><td>.183</td><td>.238</td> </tr> </table>		1	2	3	4	5	6	7	8	d	+	-	+	-	-	+	+	+		.069	.028	.109	.041	.018	.014	.249	.08		9	10	11	12	13	14	15	16	d	+	+	+	+	+	+	+	+		.032	.189	.119	.37	.386	.241	.183	.238	M1 A1	Differences ignore sign At least 3 correct
	1	2	3	4	5	6	7	8																																																	
d	+	-	+	-	-	+	+	+																																																	
	.069	.028	.109	.041	.018	.014	.249	.08																																																	
	9	10	11	12	13	14	15	16																																																	
d	+	+	+	+	+	+	+	+																																																	
	.032	.189	.119	.37	.386	.241	.183	.238																																																	
	$\bar{d} = 0.137 \quad s = 0.135 \quad n = 16$ $t = \frac{0.137 - 0}{\frac{0.135}{\sqrt{16}}} = 4.05 \text{ to } 4.07$	mldep M1 mlft A1	attempt to find \bar{d} , s Use of $\frac{s}{\sqrt{16}}$ ft (\pm) 4.06																																																						
	$df = 15$ $ cv = 2.602$	B1	cao cv p=0.00051 comp 1%																																																						
	Reject H_0 Significant evidence to suggest that odd tail feathers are less yellow than typical feathers.	A1 dep E1 dep	dep correct ts and cv(both + or both -) In context dep A1																																																						
			10																																																						

Q	Solution	Marks	Total	Comments																								
1(a)(i)	H_0 pop mean diff $\mu_d = 0$ H_1 pop mean diff $\mu_d \neq 0$ 2 tail 5%	B1		Must refer to pop mean differences or μ_d																								
	$d = K - EMC$																											
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		1	2	3	4	5																						
	d	-0.4	0.8	-0.4	-0.5	0.1																						
		6	7	8	9	10																						
		-0.8	-0.5	0	-0.6	0.1																						
	$\bar{d} = -0.22$ $s = 0.471$ $n = 10$	m1		attempt to find \bar{d} , s can be implied																								
	$t = \frac{-0.22 - 0}{\frac{0.47}{\sqrt{10}}} = -1.48$	m1 m1 A1		Use of $\frac{s}{\sqrt{10}}$ ft Method for t (\pm) 1.48 (1.46 – 1.48)																								
	$df = 9$ $cv = -2.262$ $-2.262 < -1.48$	B1		for correct cv (or $p = 0.17 > 0.05$ B1)																								
Accept H_0			*sc4 '0' ignored scores B1 M1m1m1 $\frac{s}{\sqrt{9}}$																									
<u>No significant evidence</u> to suggest that there is a difference in mean <u>measurements</u> for the two <u>devices</u> .	E1	8	correct conclusion in context																									
(ii) Assumed that differences in first ray foot measurements are normally distributed	E1		Normal distribution mentioned <u>in a sentence</u> E1																									
	E1	2	Differences in foot measurements are normal gains other E1																									

Q	Solution	Marks	Total	Comments																								
3(a)(i)	H_0 pop mean diff $\mu_d = 5$ H_1 pop mean diff $\mu_d > 5$ 1 tail 10% $d = \text{Brand} - \text{Sup Own}$	B1		H_0 pop mean diff $\mu_d = 5$ H_1 pop mean diff $\mu_d < -5$ H_1 must be consistent with d																								
	<table border="1"> <tr> <td></td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> </tr> <tr> <td>d</td> <td>+7</td> <td>+13</td> <td>-1</td> <td>+6</td> <td>-4</td> </tr> <tr> <td></td> <td>6</td> <td>7</td> <td>8</td> <td></td> <td></td> </tr> <tr> <td>d</td> <td>+16</td> <td>+1</td> <td>+5</td> <td></td> <td></td> </tr> </table>		1	2	3	4	5	d	+7	+13	-1	+6	-4		6	7	8			d	+16	+1	+5			M1		Differences (can be reversed)
		1	2	3	4	5																						
	d	+7	+13	-1	+6	-4																						
		6	7	8																								
	d	+16	+1	+5																								
	$\bar{d} = 5.375$ $s = 6.781$ $n = 8$	m1		Attempt to find \bar{d} , s can be implied by correct ts																								
	$t = \frac{5.375 - 5}{6.781 / \sqrt{8}} = 0.156$	M1		Use of $\frac{s}{\sqrt{8}}$ ft																								
		M1 m1 A1		Use of $\bar{x} - 5$ or $\bar{x} + 5$ Whole method for t (\pm)0.156 (0.15–0.16) SC $t = -4.3275$ B3																								
	$df = 7$ $ cv = 1.415$	B1		For correct consistent cv CAO																								
$0.156 < 1.415$ Accept H_0 Significant evidence to suggest that claim is correct.	E1	9	Clear correct conclusion in context of claim or jam contents																									
(ii)	Assumed that differences in jar jam contents are normally distributed and samples of jars are obtained at random.	E1		Diffs of contents / jam normal																								
		E1	2	Jars obtained at random																								