

A Level Statistics

AQA Past Exam Questions

TOPIC: Hypothesis Testing

Two Way ANOVA

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions **on paper**
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Unless otherwise stated, statistical tests should be carried out at the 5% significance level.
- When a calculator is used, the answer should be given to three significant figures unless otherwise stated.

Information

- **You may use the** booklet 'Statistical Formulae and Tables'
- There are **8** questions in this question paper. The total mark for this paper is **49**
- The marks for **each** question are shown in brackets – use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.
- Check your answers if you have time at the end.

Q	Solution	Marks	Total	Comments																																								
5(a)(i)	<p>A B C D E F G $T_A=13$ $T_B=33$ $T_C=45$ $T_D=16$ $T_E=18$ $T_F=21$ $T_G=17$ $n_A=3$ $n_B=3$ $n_G=3$</p> <p>$T = 163$ $\sum \sum x_{ij}^2 = 1705$ $N = 21$ Making $\sum \frac{T_j^2}{n_j} = \frac{73^2}{7} + \frac{54^2}{7} + \frac{36^2}{7} = 1363$</p> <p>Tea Brands $\sum \frac{T_i^2}{n_i} = \frac{13^2}{3} + \frac{33^2}{3} + \dots + \frac{17^2}{3} = 1531$</p> <p>$SS_{\text{makings}} = 1363 - \frac{163^2}{21} = 97.81$</p> <p>$SS_{\text{teas}} = 1531 - \frac{163^2}{21} = 265.81$</p> <p>$SS_{\text{Total}} = 1705 - \frac{163^2}{21} = 439.81$</p> <table border="1"> <thead> <tr> <th></th> <th>SS</th> <th>df</th> <th>ms</th> </tr> </thead> <tbody> <tr> <td>Makings</td> <td>97.81</td> <td>2</td> <td>48.91</td> </tr> <tr> <td>Tea Brands</td> <td>265.81</td> <td>6</td> <td>44.30</td> </tr> <tr> <td>Error</td> <td>76.19</td> <td>12</td> <td>6.35</td> </tr> <tr> <td>Total</td> <td>439.81</td> <td>20</td> <td></td> </tr> </tbody> </table>		SS	df	ms	Makings	97.81	2	48.91	Tea Brands	265.81	6	44.30	Error	76.19	12	6.35	Total	439.81	20		<p>M1ft</p> <p>M1ft</p> <p>M1ft</p> <p>M1ft</p>	<p>SS for makings attempt</p> <p>SS for teas attempt</p> <p>SS for total attempt</p> <p>Error SS ft (not -ve)</p> <p>Method for ms (not -ve) Condone only one correct or clear method seen ft</p>																					
	SS	df	ms																																									
Makings	97.81	2	48.91																																									
Tea Brands	265.81	6	44.30																																									
Error	76.19	12	6.35																																									
Total	439.81	20																																										
	<p>$H_0 \mu_{\text{first}} = \mu_{\text{second}} = \mu_{\text{third}}$ H_1 at least 2 of the means differ $T_s = \frac{48.91}{6.35} = 7.70$ $F_{12}^2 = 3.885 < 7.70$ Reject H_0.</p> <p>$H_0 \mu_A = \mu_B = \dots = \mu_G$ H_1 at least 2 of the means differ $= \frac{44.30}{6.35} = 6.98$ $F_{12}^6 = 2.996 < 6.98$ Reject H_0.</p>	<p>B1</p> <p>m1ft</p> <p>B1</p> <p>A1dep</p> <p>m1ft</p> <p>B1</p> <p>(A1)</p>	<p>Correct hypotheses seen once</p> <p>Method for F for makings cv correct 3.885 or $p = 0.007$</p> <p>A1 dep ts/cv or p correct</p> <p>Method for F for teas cv correct 2.996 or $p = 0.0022$</p>																																									
(ii)	<p>There is a significant difference between at least two of the making orders and between at least two of the brands</p> <p>First making is significantly preferred to the third making.</p> <p>Don't use a tea bag more than twice, preferably only once</p> <p>Brand C seems to be the favourite tea brand and Brand A the least favourite</p>	<p>E1dep</p> <p>E1</p> <p>or</p> <p>E1</p> <p>or</p> <p>E1</p>	<p>12</p> <p>2</p>	<p>A1 for Reject for both E1 In context for both dep A1 Might see in earlier conclusions</p> <p>For any two relevant comments</p>																																								
(b)(i)	<p>There is no interaction [between tea brand and making order.]</p> <p>One brand is not better/worse at particular making.</p>	<p>B1</p> <p>E1</p>																																										
(ii)	<p>The population of ratings is normal and the ratings have a common variance</p>	<p>B1</p> <p>E1</p>	<p>4</p>	<p>Normal and common variance Reference to context/ratings somewhere</p>																																								
(c)	<p>H_0 pop mean/median diff = 0 H_1 pop mean/median diff \neq 0</p> <p>Ranks</p> <table border="1"> <thead> <tr> <th>1</th> <th>2</th> <th>3</th> <th>4</th> <th>5</th> <th>6</th> <th>7</th> <th>8</th> <th>9</th> <th>10</th> </tr> </thead> <tbody> <tr> <td>9</td> <td>2</td> <td>5</td> <td>.</td> <td>1</td> <td>3½</td> <td>7</td> <td>6</td> <td>8</td> <td>3½</td> </tr> <tr> <td>1</td> <td>8</td> <td>5</td> <td></td> <td>9</td> <td>6½</td> <td>3</td> <td>4</td> <td>2</td> <td>6½</td> </tr> <tr> <td>-</td> <td>+</td> <td>-</td> <td></td> <td>-</td> <td>-</td> <td>-</td> <td>-</td> <td>-</td> <td>+</td> </tr> </tbody> </table> <p>$T_- = 9 + 5 + 1 + 3\frac{1}{2} + 7 + 6 + 8 = 39\frac{1}{2}$ $T_+ = 2 + 3\frac{1}{2} = 5\frac{1}{2}$</p> <p>ts $T_+ = 5\frac{1}{2}$ cv = 6 $T_+ < 6$ Reject H_0 There is a significant difference the brands – C preferred</p>	1	2	3	4	5	6	7	8	9	10	9	2	5	.	1	3½	7	6	8	3½	1	8	5		9	6½	3	4	2	6½	-	+	-		-	-	-	-	-	+	<p>M1</p> <p>m1</p> <p>A1</p> <p>B1</p> <p>E1dep</p>	<p>5</p>	<p>ranks – any effort</p> <p>totals of ranks +/- correct</p> <p>cv correct and correct comparison with lower ts in context ts and cv correct and hypotheses effort</p>
1	2	3	4	5	6	7	8	9	10																																			
9	2	5	.	1	3½	7	6	8	3½																																			
1	8	5		9	6½	3	4	2	6½																																			
-	+	-		-	-	-	-	-	+																																			

5(a)(i)		O	M	R	Tot		
	A	42	29	19	90		
	B	37	33	24	94		
	C	24	29	18	71		
	D	25	22	13	60		
	Tot	128	113	74	315		
	$\Sigma x^2 = 9019$						
	Between Models SS						
		$\left(\frac{90^2 + 94^2 + 71^2 + 60^2}{3} \right) - \frac{315^2}{12} = 256.92$				M1	method between models SS
	Between Campers SS						
		$\left(\frac{128^2 + 113^2 + 74^2}{4} \right) - \frac{315^2}{12} = 388.5$				M1	method between campers SS
	Total SS = $9019 - \frac{315^2}{12} = 750.25$						
		Source	SS	DF	MS		
		Models	256.92	3	85.64	M1	method Error SS
		Campers	388.50	2	194.25	B1	df 3,2,6
	Error	104.83	6	17.47	m1	MS - their df	
	Total	750.25	11				
H ₀ : no difference between models							
F = 85.64/17.47 = 4.90							
cv F _[3,6] is 4.757 reject H ₀ - not all models take same time to pitch on average							
					M1	method for F - their positive SS and df	
					B1	4.757 and 5.143 (2dp)	
H ₀ : no difference between campers							
F = 194.25/17.47 = 11.1							
cv F _[2,6] is 5.143 reject H ₀ - not all campers take same time to pitch a tent on average							
					A1√	conclusion - must be compared with upper tail of F	
					M1	method for F - their positive SS and df	
					A1	4.90 (4.85 - 4.95) and 11.1(11.0 to 11.2)	
					A1√	conclusion- must be compared with upper	
					A1√	with upper tail of F	
						both conclusions in context – needs both previous A1√ marks	

Q	Solution	Marks	Total	Comments
5(a)(ii)	D appears to take least time to pitch but this could be because it is always pitched last after practice on other models.	B1 E1		D CAO because always pitched last

	Solution	Marks	Total	Comments																				
5(b)	$H_0 \mu_{Run} = \mu_{Cycle} = \mu_{Swim}$ H_1 at least 2 of the means differ $T_{Male} = 26.0$ $T_{Female} = 23.6$ $n_{Male} = 3$ $n_{Female} = 3$ $T_{Run} = 14.7$ $T_{Cycle} = 17.5$ $T_{Swim} = 17.4$ $n_{Run} = 2$ $n_{Cycle} = 2$ $n_{Swim} = 2$ $T = 49.6$ $\sum \sum x_{ij}^2 = 413.78$ $N = 6$ Total SS $413.78 - \frac{49.6^2}{6} = 3.753$ Sex SS $\frac{26.0^2}{3} + \frac{23.6^2}{3} - \frac{49.6^2}{6} = 0.96$ Sport SS $\frac{14.7^2}{2} + \frac{17.5^2}{2} + \frac{17.4^2}{2} - \frac{49.6^2}{6} = 2.523$																							
		M1		Total SS method (can be implied in table)																				
		M1		Sex SS method																				
		M1		Sport SS method																				
	<table border="1"> <thead> <tr> <th></th> <th>SS</th> <th>df</th> <th>MS</th> </tr> </thead> <tbody> <tr> <td>Sex</td> <td>0.96</td> <td>1</td> <td>0.96</td> </tr> <tr> <td>Sport</td> <td>2.523</td> <td>2</td> <td>1.26</td> </tr> <tr> <td>Error</td> <td>0.27</td> <td>2</td> <td>0.135</td> </tr> <tr> <td>Total</td> <td>3.753</td> <td>5</td> <td></td> </tr> </tbody> </table>		SS	df	MS	Sex	0.96	1	0.96	Sport	2.523	2	1.26	Error	0.27	2	0.135	Total	3.753	5		M1		Error SS ft (not -ve)
	SS	df	MS																					
Sex	0.96	1	0.96																					
Sport	2.523	2	1.26																					
Error	0.27	2	0.135																					
Total	3.753	5																						
		B1		Error df correct $v = 2$																				
		m1		Method for MS ft (dep SSe)																				
		m1		Method (dep prev Ms) for F (sex/error or sport/error) Not -ve																				
	$F = \frac{1.26}{0.135} = 9.35$ $F_2^2 = 19.0$	A1		Sports F correct 9.2-9.5																				
	$9.35 < 19$ Accept H_0	B1		cv correct CAO or $p = 0.117$																				
	There is no significant evidence of a difference in mean training times for the the 3 sports.	E1	10	Correct conclusion in context, ft small arithmetic error in F																				
	Total		22																					

(b)

source	SS	DF	MS
alcohol	9348	2	4674
weights	7980	3	2660
residual	3214	6	535.67
total	20542	11	

H_0 : no difference between amounts of alcohol

$$F = \frac{4674}{535.67} = 8.73$$

reject H_0 : significant evidence differences in mean times to do Sudoku between groups drinking different amounts of alcohol

H_0 : no difference between weights

$$F = \frac{2660}{535.67} = 4.97$$

$$\text{c.v. } F_{[3,6]} = 4.757$$

reject H_0 : significant evidence differences in mean times to do Sudoku between groups of different weights

B1

M1

m1

A1

B1

A1✓

6

2, 3, 6 df
method for all MS (including method for residual SS), their df

method for F (either) – their figures

8.73(8.72~8.73) and 4.97(4.96~4.97)

5.143(5.14~5.15) and 4.757(4.75~4.76)

both conclusions – their figures – must be compared with upper tail of F