A-LEVEL

Statistics

Statistics 4 – SS04

Mark scheme

6380

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Version/Stage: 1.0 Final

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this Mark Scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Solution	Marks	Total	Comments
1	$H_0 : p = 0.15$ $H_1 : p > 0.15$	B1		Both. oe in numbers (μ=18) or in words
	Using Numbers $X \sim B(120,0.15) \approx N(18,15.3)$	B1 B1		B1 for mean 18 cao B1 for correct expression for variance or 15.3 cao or SD awrt 3.91
	TS = $\dfrac{29-18}{\sqrt{15.3}} = 2.81$	M1 A1		Their mean/SD condone small slip Ignore sign but needs square root. 2.75 ~ 2.9
	or TS = $\dfrac{28.5-18}{\sqrt{15.3}} = 2.68$	(M1) (A1)		2.6 ~ 2.75 Note: wrong CC (29.5) gives 2.94 for M1A0
	or *Using Proportions* Use of N(0.15, 0.00106)	(B1) (B1)		B1 for mean 0.15 cao B1 for correct expression for variance or (0.0010 ~ 0.0011) or SD (0.031~0.034)
	$\hat{p} = \dfrac{29}{120} = 0.242$ TS = $\dfrac{0.242-0.15}{\sqrt{\dfrac{0.15\times0.85}{120}}} = 2.81$	(M1) (A1)		Their mean/SD condone small slip (eg use of $\hat{p}$ in denominator →z = 2.34). Ignore sign but needs square root. 2.75 ~ 2.9
	Critical value is 2.3263	B1		2.32 ~ 2.33 **or** p=0.0025 (0.0015 ~ 0.003) from TS=2.81 (using 29) **or** p=0.0037 (0.0029 ~ 0.005) from TS=2.68 (using 28.5) **or** p=0.0016 (0.001 ~ 0.002) from TS=2.94 (using 29.5) **or** if no CV seen and p not one of these, B1 for their normal or binomial p-value (< 0.5) compared to 0.01.
	Reject H_0 at the 1% level	A1dep		Requires correct TS **or** 2.94 and correct critical z value **or** their correct p compared with .01. Must correctly reject H_0
	There is evidence that the **percentage (/proportion)** of **customers (/ those aged 25 years)** or under is **higher** for the new savings account.	E1dep		In context. Dependent on all previous marks, except first B1..
			8	
	Total		**8**	

Q	Solution	Marks	Total	Comments
2 (a)	B(240,0.0025) → Po(0.6)	M1 A1		Attempt at **Poisson** approximation Mean n× 0.0025 = 0.6
	P(3 or more) = 1 − 0.9769	m1		Attempt to find P (3 or more) from either 1 - 0.9769 or 1 - 0.9966 (= 0.003 ~ 0.004)
	= 0.0231	A1		0.023 ~0.024 **Note** Exact binomial (0.0229) or normal approx gets 0/4
			4	
(b)	Using $45 \pm z\sqrt{45}$ z=1.96 used in a CI 45 ± 13.15 or (31.85, 58.15)	M1 B1 A1		Any z cao (13.1~13.2) (31.8~31.9, 58.1~58.2)
			3	
		Total	**7**	

Q	Solution	Marks	Total	Comments
3(a) (i)	$H_0 : \lambda = 2.8$ $H_1 : \lambda > 2.8$	B1		For both. Allow μ or "rate".
	Find $P(X \geq 4)$ from Poisson tables	M1		*Attempt* to calculate $P(X \geq 4)$ from Po(2.8). Besides 1 – 0.6919, allow for $P(X > 4)$ = 1 – 0.8477 (= 0.1523) **or** $P(X = 4)$ = 0.8477-0.6919 (=0.1558) **or** just 0.8477 or 0.6919
	= 1 – 0.6919 = 0.308(1)	A1dep		1 – 0.6919 or 0.308 ~ 0.309 **or** 0.691~0.692 *if* comparing with 0.90
	This is > 0.10 so do not reject H_0. **Claim not supported**. OR There is **no** evidence that the rate of shark **attacks** has **increased**.	M1dep E1dep		Acc H_0 + correct Poiss prob with 0.10 oe Correct P-value and 0.10, including conclusion in context. Dependent on M1A1M1.
			5	
(ii)	$\bar{x} = 38 \quad s = 15.556$	B1		For 38 cao and s_{n-1} = 15.5 ~ 15.6 or s_n=13.472 (13.4 ~ 13.5) (ignore labels)
	$H_0 : \mu = 52$ $H_1 : \mu < 52$	B1		Both (or in words)
	$(t =) \dfrac{38-52}{15.556/\sqrt{4}}$ **OR** $(t =) \dfrac{38-52}{13.472/\sqrt{3}}$	M1		M1 for use of $\dfrac{s_{n-1} \text{ or } s_n}{\sqrt{4} \text{ or } \sqrt{3}}$ in test statistic formula
	$= -1.80$	m1 A1		Correct formula, ignore sign. Allow z=. -1.81 ~ -1.79. Ignore sign
	Critical value $t_3 = -1.638$	B1 B1		3 df in *t*-test used (implied by 1.638 or 2.353) -1.63 ~ -1.64 (beware use of 5% normal CV). Sign consistent in use with TS **or** *p* = 0.0848 (0.084 ~ 0.086)
	Reject H_0 at 10% level. There is evidence to **support** claim that shark **attacks** on average are **closer** to shore.	E1dep		Correct TS & critical t (signs consistent) OR correct *p*-value and 0.10. Concl in context. Dependent on all previous marks except B1 for hyps
	Note 1 z test (eg using p = 0.036) gets max B1B1M1M1A1B0B0E0 for max 5/8 **Note 2** Equivalent solutions using **one**-sided confidence intervals or decision intervals can get full marks from 50.7<52 OR 38<39.3 so reject H_0. Use of **two**-sided intervals 56.3>52 OR 38>33.7 so accept H_0 will lose at least the final B1E1.			
			8	
(b)	**Both** statements **agree** with the conclusions in (a)(i) AND (a)(ii).	B1		No ft (i.e. needs (i) not rej H_0 (ii) rej H_0)
	However, failure to reject H_0 does NOT mean that H_0 is "proved". The first statement is too strong. The second statement is also too positive.	B1		B1 for recognising that statements are too positive/definite. Allow correct reference to a relevant type of error that may have been made (type II then type I) or other reasonable explanation of why conclusion(s) should be treated with caution. NIS.
			2	
		Total	**15**	

Q	Solution	Marks	Total	Comments
4 (a)	E(T) = 78 + 126 = 204 V(T) = $5.8^2 + 7.4^2$ = 88.4	B1 M1 A1		Cao Awrt 88.4 isw. Ignore units here. **SC** If M0A0**,** SD = 9.4(0) B1
			3	
(b)	Weekly profit (X) = $0.22W_A + 0.15W_B$ E(X) = 0.22×78 + 0.15×126 = 36.06 V(X) = $0.22^2 \times 5.8^2 + 0.15^2 \times 7.4^2$ (= 1.628 + 1.232 = 2.860) SD(X) = sqrt (2.860) = 1.69	M1 A1 m1 A1		Attempt to use this linear comb. (PI) Correct expression or 17.16+18.90 seen . AG Method for variance. May be implied by $1.276^2 + 1.110^2$. Condone missing squares on 5.8 and 7.4. Completely correct expression for SD seen. AG
			4	
(c) (i)	E(L) = 17.16 – 18.9 (= –1.74) SD(L) = SD(X) = 1.69 $P(L > 0) = P\left(Z > \frac{1.74}{1.69}\right) = P(Z > 1.0296)$ = 1 – 0.8484 = 0.1516	B1 BF1 M1 m1 A1		Ignore sign Same as in (b) or may start again. Standardising, clearly using mean &SD Using tables. Allow for 0.8484. 0.151 ~ 0.152 (z = 1.03 in tables gives 0.1515)
	Note E(L) = 1.74, P(L > 0) = 0.1516 is equivalent E(L) = 1.74, P(L < 0) = 0.8484 gets B1BF1M1m1A0 for max 4/5			
			5	
(ii)	It is the probability that, in a particular week, the pet shop will make a **greater profit** from **Appydog** sales than from Boneybites sales.	E1		oe
			1	
(d)	$H_0: \mu = 78$ $H_1: \mu > 78$ $z = \frac{82 - 78}{5.8/\sqrt{8}} = 1.95(06)$ CV = 2.3263 **or** p (= 1 – 0.9744) = 0.0256 Do not reject H_0 at the 1% level. No evidence to suggest **sales increased** OR **campaign successful**. (oe)	B1 M1 A1 B1 E1dep		Both Correct expression. Ignore sign and label 1.94 ~ 1.96 2.32 ~ 2.33. (± sign consistent with TS). **or** 0.025 ~ 0.026 Correct conclusion in context, comparing correct TS with CV **or** correct p-value with 0.01. Dependent on M1A1B1.
	Note Using CI: (B1) $82 - k \times \frac{5.8}{\sqrt{8}}$ M1 2.32 ~2.33 B1 LCL = 77.2 A1 77.2 < 78 so accept H_0 + conclusion E1 Using DI: (B1) $78 + k \times \frac{5.8}{\sqrt{8}}$ M1 2.32 ~2.33 B1 UDL = 82.8 A1 82.8 > 82 so accept H_0 + conclusion E1			
			5	
		Total	18	

Q	Solution	Marks	Total	Comments
5 (a) (i)	0.10 or 10%	B1		oe
			1	
(ii)	P(≥ 1 exclude μ) = 1 – P(None exclude μ) $= 1 - (0.9)^5 = 1 - 0.59049$ = 0.40951	M1 M1 A1		Any p^5 with $0<p<1$ seen For $1 - (0.9)^5$ or $1 - (0.1)^5$ (= 0.99999) 0.4 ~ 0.41
	Alternative Using B(5, 0.1) table P(≥ 1) = 1 – 0.5905 **or** P(> 1) = 1 – 0.9185 (= 0.0815) = 0.4095	 (M1) (M1) (A1)		 Seen or implied Either expression seen 0.4 ~ 0.41
			3	
(b) (i)	$10.280 \pm t_{5;0.05}\dfrac{0.021}{\sqrt{6}}$ with $t = 2.015$ $= 10.280 \pm 0.017(275)$ giving limits (10.263,10.297)	M1 B1 M1 A1		Correct form including $\sqrt{6}$ and use of *t* distribution or z = 1.64 ~ 1.65 Condone 10.325 ± (…) 2.01 ~ 2.02 Completely correct expression evaluated. 10.28cao ± (0.017 ~ 0.0175) may be implied by correct limits not necessarily to 3dp. Both limits to 3dp. No isw here. Condone truncation to 10.262.
			4	
(ii)	**10.325** is **outside** (or **above**) the interval So new programme seems effective (or mean time decreased)	B1 B1dep		Needs their CI basically correct. Condone small slip but CI must be below 10.325. Need only refer to upper limit. Correct conclusion and reason. Dependent on previous B1 **Note.** These last 2 marks are available after using normal in part (i) which gives (10.266, 10.294)
			2	
		Total	**10**	

Q	Solution	Marks	Total	Comments
6 (a) (i)	No. of cherries in the cake (C) ~ Po(15)	B1		Poisson 15. Allow for B(450 , $\frac{1}{30}$) (→0.737)
	P(C>12) = 1- 0.2676	M1		Attempt at P(C>12)from Po(15). Allow also P(C≥12) = 1 – 0.1848 (= 0.8152)
	= 0.7324	A1		0.732 ~ 0.733
			3	
(ii)	No. of currants in the cake (U) ~ Po(225)	B1		Poisson 225
	Approximated by N(225,225)	M1		For Normal approximation to Poisson.
	P(U ≤ 250) = P(U < 250.5)			
	$=P\left(Z < \frac{250.5-225}{\sqrt{225}}\right)$	m1		For standardisation. Condone missing/wrong CC. Ignore sign.
	$= P(< 1.7)$	A1		A1 for completely correct expression. May be implied by z = 1.7.
	= 0.9554(3)	A1		0.955 ~ 0.956
				Special Case Exact Poisson→0.9535 (0.953 ~ 0.954) B2
	Note Missing/wrong CC gives answers 0.952 and 0.949 respectively for B1M1m1 max 3/5			
			5	
(iii)	(1-(ii)) × (1-(ii))	M1		For p^2 where p=1-answer in (ii) or (p = 1-answer from starting again with sultanas) **and** 0 < p < 1
	= 0.0446^2 = 0.00199	AF1		0.0019 ~ 0.0021 ft their p
			2	
(b) (i)	n = 75	B1		Cao. Award in (b)(ii) if n not given here
	No. of cherries in cake of $45cm^3$ (S) ~ Po(1.5)	M1		M1 for Po(1.5)
	Then p = P(S = 0) = $e^{-1.5}$	m1		Attempt at P(S=0) from Po(1.5)
	= 0.223(1)	A1		0.223 correctly derived. Requires $e^{-1.5}$ seen (oe).
			4	
(ii)	75 × 0.223 = 16.73	B1		75 × 0.223 **or** 16.7 ~ 16.8 isw
			1	
(iii)	If not stirred enough, cherries would tend to appear in 'clumps'. So more cupcakes would be expected to be cherry-free	E1		oe "*Not distributed at random*" is not enough. Needs idea of "clumping".
	Hence would expect to **increase** E(G).	E1		Disallow if with *incorrect* or *nonsense* justification. Allow if with *insufficient* or *no* justification Allow other **convincing** arguments.
			2	
		Total	**17**	