A Level Statistics
AQA Past Exam Questions

TOPIC: Hypothesis Testing
Two Sample Mean

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have retrievable mathematical formulae stored in them.

Instructions

• Use **black** ink or ball-point pen.

• If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.

• **Fill in the boxes** at the top of this page with your name, centre number and candidate number.

• Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.

• Answer the questions ***on paper***

• You should show sufficient working to make your methods clear. Answers without working may not gain full credit.

• Unless otherwise stated, statistical tests should be carried out at the 5% significance level.

• When a calculator is used, the answer should be given to three significant figures unless otherwise stated.

Information

• ***You may use the*** booklet 'Statistical Formulae and Tables'

• There are **17** questions in this question paper. The total mark for this paper is ***125***

• The marks for **each** question are shown in brackets – use this as a guide as to how much time to spend on each question.

Advice

• Read each question carefully before you start to answer it.

• Try to answer every question.

• Check your answers if you have time at the end.

• If you change your mind about an answer, cross it out and put your new answer and any working underneath.

• Check your answers if you have time at the end.

AQA_SS05_JUNE_2018_3b

(b) (i)	$S_p^2 = \dfrac{9\times0.9615^2 + 11\times1.21^2}{20} = 1.22$	M1		Numerator of formula correct using "*their*" s_A or s_A^2 (=0.9244)
	AG	A1	2	CSO with full correct working
(ii)	H_0: $\mu_B = \mu_A + 0.2$ H_1: $\mu_B > \mu_A + 0.2$	B1		OE Other suffices must be clearly defined.
	$\bar{x}_A = 30$	B1		Or 3000
	t.s. $= \dfrac{30.9-30-0.2}{\sqrt{1.22\left(\frac{1}{10}+\frac{1}{12}\right)}} = 1.48$	M1 M1 A1		Numerator correct using *their* $\bar{x}_A$ Denominator correct AWRT 1.48 (allow rescaling to cm for M1M1A1)
	c.v. $t_{20} = 1.325$ or $p = 0.0773 < 0.1$	B1		correct c.v. or p-value correct, compared to 0.1
	$(1.48 > 1.325)$ Reject H_0.	A1dep		Correct conclusion dependent on all values correct.
	There is significant evidence (at the 10% level) that rolls of baking parchment of Brand B are more than 20 cm longer on average than those of Brand A.	E1dep	8	In context

AQA_SS05_JUNE_2016_6b

6(b)	H_0: $\mu_{car} = \mu_{alt\ means} + 1$ H_1: $\mu_{car} > \mu_{alt\ means} + 1$	B1 B1		B1: an inequality in μ and "1" B1: all correct; other suffices must be clearly identified – allow "c" and "a".
	$\bar{x}_{car} = 26.1$ $\bar{x}_{alt.\ means} = 24.225$ $S_{car} = 2.09$ $S_{alt\ means} = 1.74$ $s_p^2 = \dfrac{10\times2.09^2 + 7\times1.74^2}{17} = 3.82$	B1 B1 M1		B1; either mean (cao, 24.2 ~24.3) B1: either s (2.09 ~2.10, 1.73~ 1.74) $s_p^2 = 3.81617\ldots$ $s_p = 1.9535\ldots$ NMS award M1 for value used in range 3.80 ~ 3.84
	$t = \dfrac{26.1 - 24.225\ (-1)}{\sqrt{3.816\times(\frac{1}{11}+\frac{1}{8})}} = 0.964$	M1 M1 A1		M1: numerator M1: denominatorA1: awfw 0.9 ~ 1.0 ; must have gained all M's.
	cv $t_{17} = 1.740$	B1		cao, accept 1.74 , condone ±
	$0.964 < 1.740$ or $p = 0.174 > 0.05$; accept H_0	A1dep		Dep A1 for t.s. and B1 for positive c.v. p: awfw 1.66 ~ 1.90 (0.1743..)
	No significant evidence at the 5% level that male students who travel to college by car have a BMI which is, on average, more than 1 kg/m^2 greater than those who travel to college by alternative means of transport.	E1 dep	11	o.e. conclusion in context dep previous A1
	Total		**20**	

6(b) Alt 1 for lower tail

ts $= \dfrac{24.225 - 26.1\ (+1)}{\sqrt{3.816\times(\frac{1}{11}+\frac{1}{8})}} = -0.964$ M1 M1 M1A1 ; $t_{17} = \pm1.740$ B1

$-0.964 > -1.74$; accept H_0 A1 dep (nb signs resolved) ; E1 dep as on MS

Alt 2

Use of p value and if no intermediate evidence seen: B1 B1 hypotheses as on MS ; $p = 0.166 \sim 0.190$ (0. 1743…) implies B1 B1 M1 M1 M1 A1 (outside this range and they lose all 6 marks) ;comparing 0.174… > 0.05 B1 and accept H_0 A1 (dependent on previous A1 and B1) ; E1 dep as on MS.

AQA_SS05_JUNE_2017_5b/c

5(b)	$H_0: \mu_x = \mu_y$ $H_1: \mu_x > \mu_y$	B1		both hypotheses
	$\bar{x} = 3.88 \quad \bar{y} = 3.49$	B1		either 3.88 ; 3.48 ~3.50 (3.4911…)
	$S_p{}^2 = \frac{5 \times 0.582^2 + 8 \times 0.683^2}{13} = 0.417$	M1		M1: use of correct formula (0.417…; s_p = 0.6459..)
	t.s. $= \frac{3.88-3.49}{\sqrt{0.417\left(\frac{1}{6}+\frac{1}{9}\right)}}$	M1 M1		numerator (accept (3.49 – 3.88)) denominator (ft on $s_p{}^2$ if M1 earned)
	= 1.142	A1		1.13 ~ 1.15 (± to agree with their numerator)
	c.v. $t_{13} = \pm 1.35$ or p - value = 0.137 > 0.1	B1		±1.35 or p-value= 0.13 ~ 0.14(0.1371…) compared with 0.1
	1.142 < 1.35 accept H_0	A1dep		Dep on A1 for t.s. , B1 for c.v. or p - value and correct signs. (accept -1.35 < -1.142)
	Evidence at the 10% level that Emily's suspicion is not supported.	E1dep		Conclusion in context dep on previous A1dep oe e.g. Alt: insufficient evidence at 10% level that babies born in the summer months are on average heavier than those born at other times of the year.
			9	
5(c)(i)	H_0 has been accepted so a Type II error might have been made.	E1		stating eg Type II error is when H_0 is accepted – H_0 must be accepted in (b) s.c. B1 for Type I error etc… if H_0 is rejected in (b)
(ii)	It was incorrectly concluded that there was no difference between the mean weight of babies born in the summer months and the mean weight of babies born at other times of the year. or Mean weight of babies born in the summer months is greater than the mean weight of babies born at other times of the year. or H_0 incorrectly accepted; Emily's suspicion should have been justified.	E1dep, E1 dep		E1 explanation of a Type II error eg H_0 accepted when it should have been rejected ; dep E1 in c(i) E1 all correct – must have "mean " or "average" or "Emily's suspicion"
			3	

AQA_SS05_JUNE_2015_2

Q	Solution	Marks	Total	Comments
2(a)	$\bar{x}_{2012} = 264 \quad \bar{x}_{2010} = 256.4$	B1,B1		264 , 256~256.5
	$\sigma^2_{2012} = 551 \; \sigma^2_{2010} = 660$ $(s^2_{2012} = 558 \; s^2_{2010} = 669)$	B1, B1		Accept either σ^2 or s^2 but must be consistent. awfw 551~558, 660~670
			4	
(b)	(i) $H_0: \mu_{2012} = \mu_{2010}$ $H_1: \mu_{2012} > \mu_{2010}$	B1		both
	t.s. $z = \frac{264-256.4}{\sqrt{\left(\frac{558}{90}+\frac{669}{75}\right)}}$	M1 M1		M1: numerator ; accept 256.4 –264 M1: denominator ; allow use of (consistent) s^2 or σ^2 (ft only on a small numerical slip)
	= 1.954	A1		1.9 ~ 2.1; accept ±
	c.v. $z = 1.6449$	B1		1.64 ~ 1.65 ; accept ± or p = 0.0253 (0.024~ 0.026) compared with 0.05
	1.954 > 1.6449 reject H_0 Evidence at the 5% level that the mean weight of chicks was greater in 2012 than in 2010.	E1		Comment in context; All working correct with consistent signs. Accept "mean weight greater in 2012" oe
			6	
	(ii) sample sizes are large so means are approx. normally distributed due to Central Limit Theorem.	E2		E1 large samples E1 CLT
			2	
(c)	This would mean concluding that the mean weight of chicks was greater in 2012 than in 2010 when in fact the means were the same.	E2		Statement in context s.c. E1 – no context: eg Type 1 error is when H_0 is rejected when it is true.
			2	

AQA_SS05_JUNE_2014_2

2(a)	$H_0: \mu_{females} - \mu_{males} = 1.5cm$ $H_1: \mu_{females} - \mu_{males} > 1.5cm$	B1 B1	**2**	B1 : an inequality and 1.5 B1 : both correct
(b)	t.s. $= \frac{(8.54-6.28-1.5)}{\sqrt{\left(\frac{0.6^2}{10}+\frac{0.6^2}{8}\right)}}$ $= 2.67$	M1 M1 A1		numerator , allow "8.54 – 6.28" seen denominator awfw 2.67~2.675
	z = 2.3263	B1		Ignore sign , *p* - value : 0.00378 ~ 0.00379
	2.67 > 2.3263 - reject H_0	A1		comparison of correct cv with correct t.s; both + ve or both - ve.
	Sufficient evidence at 1% level to suggest that the mean length of female toads is more than 1.5cm greater than the mean length of male toads.	E1dep		Statement in context – dependent on previous A1.
			6	
			8	
	Notes: Hypotheses: must use μ or population mean. Allow *f* and *m* as suffices but other suffices only if clearly assigned.			

AQA_SS05_JUNE_2013_2

2	$H_0: \mu_A = \mu_B + 24$ $H_1: \mu_A > \mu_B + 24$	B1 B1		(s.c. B1 for both $H_0: \mu_A = \mu_B$ and $H_1: \mu_A > \mu_B$)
	$\bar{x}_A = 473 \quad \bar{x}_B = 438$	B1		B1 both means; awfw 472 ~ 473 and 438 ~ 439
	test statistic $z = \frac{473-438-24}{\sqrt{\frac{7^2}{6}+\frac{10^2}{8}}}$	M1 M1		M1 Numerator (allow (473 – 438) or (438 – 473 – 24)) M1 Denominator
	= 2.29	A1		A1 awfw 2.25 ~ 2.45
	cv 5% level 1-tail test $z = 1.6449$	B1		B1 awfw 1.64 ~ 1.65 (condone ±)
	2.42 > 1.6449 reject H_0	A1		dep A1 for ts (consistent with hypotheses) and B1 for cv
	Evidence at the 5% level to support Nasreen's belief.	E1	9	Correct comment in context dep. on previous A1 - must mention mean or average and some element of doubt. eg Some evidence that **boxes** of eggs from Alaric are **more than 24gm** heavier **on average** than those from Belinda
	Total		**9**	

AQA_SS05_JUNE_2012_4aii/b

(ii)	$\bar{x}_1 = 648.6$ $\bar{x}_2 = 619.86$	B1		B1 648.6 (648 ~ 649) and 619.86 (619.5 ~ 620)
	Pooled variance estimate $s_p^2 = (3742.49\text{x}9 + 4716.14\text{x}6)/15$ $= 4131.95$	M1		M1 method for pooled variance
	H_0: $\mu_1 = \mu_2$	B1		B1 one hypothesis correct
	H_1: $\mu_1 > \mu_2$	B1		B1 both hypotheses correct - don't penalise the same error twice
	$t = \dfrac{(648.6 - 619.86)}{\sqrt{4131.95(1/10 + 1/7)}}$	M1 M1		M1 method for numerator M1 method for denominator -
	$= 0.907$	A1		A1 0.907 (0.9 ~ 0.91) - ignore sign
	c.v. t_{15} is 1.753	B1 B1		B1 15 df B1 1.753 - ignore sign
	Accept H_0 i.e. no significant evidence of a reduction in Saturday takings after October 2011	A1√ A1√	11	A1√ accept H_0 - must be compared with correct tail of t A1√ conclusion in context 0 for contradiction (or p = 0.189 compared with 0.05)
4(b)(i)	H_0: $\mu_2 = \mu_1 + 50$ H_1: $\mu_2 > \mu_1 + 50$	B1 B1	2	B1 1 correct hypothesis B1 both correct - only penalise the same mistake once
(ii)	801,887,1013,884,964,1014,1146	M1 A1	2	M1 method A1 accuracy - allow one slip
(b)(iii)	critical value of t_{15} is 2.602	B1		B1 2.602
	reject H_0, conclude total takings will be increased by more than £50.	B1 B1	3	B1 conclusion (M implied) B1 in context must be compared with t- values (or p=0.0000936)

AQA_SS05_JUNE_2011_5aii

(ii)	pooled variance estimate $s_p^2 = \frac{(244.125 \times 7 + 316.271 \times 6)}{13}$ $= 277.423$	M1		method for pooled variance
	$H_0 : \mu_{ABC} = \mu_{XYZ} + 10$	B1		one hypothesis correct
	$H_1 : \mu_{ABC} > \mu_{XYZ} + 10$	B1		both hypotheses correct – don't penalise the same error twice
	$t = \frac{(205.125 - 192.429 - 10)}{\sqrt{277.423\left(\frac{1}{8} + \frac{1}{7}\right)}}$	M1 M1		method for numerator method for denominator allow $\frac{s_A^2}{8} + \frac{s_B^2}{7}$
	$= 0.313$	A1		0.313 (0.31 ~ 0.32); ignore sign
	c.v. t_{13} is 1.35	B1		1.35; ignore sign
	accept H_0 no significant evidence to show that *XYZ* couriers are more than 10 minutes faster than *ABC* couriers *P = 0.3797 (0.379 ~ 0.4)*	A1F A1F	9	accept H_0 – must be compared with correct tail of *t* – disallow contradiction conclusion in context – needs previous A mark

AQA_SS05_JUNE_2010_2

2(a)	$\bar{x} = 76.928 \quad s_x = 2.588896$ $\bar{y} = 73.0625 \; s_y = 2.243045$ H_0: $\mu_x = \mu_y$ $\quad H_1$: $\mu_x \neq \mu_s$	B1 B1 B1		B1 76.9 (76.9~77), 73.1 (73~73.1) 2.59 (2.58~2.6), 2.24 (2.24~2.25) B1 one correct hypothesis – generous B1 both correct – ungenerous
	pooled variance estimate $s^2 = (6 \times 2.588896^2 + 7 \times 2.243045^2)/13$ $= 5.80254 \quad (s = 2.4088)$	M1 m1		M1 attempt at pooled variance m1 correct method for pooled variance
	$t = \frac{76.928 - 73.0625}{\sqrt{5.80254\left(\frac{1}{7} + \frac{1}{8}\right)}}$ $= 3.8655/1.2467$ $= 3.10$	M1m1 A1 B1B1 A1√		M1 difference of means/their standard deviation m1 correct method for *t* A1 3.10 or –3.10 (3.09 ~ 3.11)
	c.v. $t_{13} = \pm 3.012$ Reject H_0. Conclude that mean water temperature after 5 hours for flask A is different from (higher than) for flask B	A1√	12	B1 13 df B1 3.012 or 3.01 ignore sign A1√ conclusion – must be compared with correct tail of *t* A1√ in context – requires previous A1√
(b)	Conditions not controlled e.g. background temperature, amount of water in flask. Conditions may differ between first 7 days and last 8 days.	E1 E1	2	E1 conditions not controlled E1 order of experiments not randomised or balanced one mark for any sensible point
	Total		**14**	

AQA_SS05_JUNE_2008_3b

(b)	$\bar{x}_B = 69.8429 \quad \bar{x}_A = 55.7333$			
	pooled variance estimate, s_p^2 $\dfrac{6\times 9.1354^2 + 5\times 11.030^2}{6+5} = 100.852$	M1		method for pooled variance
	$s_p = 10.043$ $H_0: \mu_B = \mu_A \quad H_1: \mu_B > \mu_A$	B1		both hypotheses - needs μ or population
	$t = \dfrac{69.8429 - 55.7333}{10.043\sqrt{\frac{1}{7}+\frac{1}{6}}}$	M1		method for t - their pooled variance allow if $\frac{s_x^2}{7} + \frac{s_y^2}{6}$ used for variance
		m1		correct method for t - ignore sign
	$= 2.53$	A1		2.53 (2.52 ~ 2.53) - ignore sign
	c.v. t_{11} is 1.796	B1 B1√		11df 1.796 - ignore sign, their df
	reject H_0, significant evidence that mean speed has been reduced after introduction of speed cameras.	A1√		conclusion - needs one sided t-test plus +ve ts compared with +ve cv or –ve ts compared with –ve cv
		A1√	9	in context - allow arithmetic errors, incorrect t-value, 2-sided test.

AQA_SS05_JUNE_2009_5b

5(cont) (b)	$H_0: \mu_p = \mu_s + 5$ $H_1: \mu_p < \mu_s + 5$	B1		Hypotheses
	$z = \dfrac{19.7333 - 15.2143 - 5}{\sqrt{(0.95^2/6 + 0.65^2/7)}}$	M1 M1		Method for variance Method for z - their variance
	$= -1.05$	A1		–1.04 ~ –1.06 - ignore sign
	c.v. –1.2816	B1		–1.28 ~ –1.282 - ignore sign
	Accept H_0, ie accept mean breaking strength of premium line is at least 5kg greater than that of standard line	A1√ A1√	7	Accept H_0 - must be compared with correct tail of z – needs both M marks Conclusion in context – needs previous A1√
	p-value 0.148 (0.146 ~ 0.149) compare with 0.1			*If t used, maximum B1M0M1A0B1(for 1.363)A0A0*

(b)	H₀: $\mu_M - \mu_A = 1$ H₁: $\mu_M - \mu_A > 1$	B1 B1		μ_M, μ_A reversed, lose first B1 and last A1 } or equivalent
	CV of $z = 1.6449$	B1		If $H_1 \neq$ must have 1.96 accept 1.64, 1.645 or $P(Z>1.94)=0.2619$
	sample value of $z = \dfrac{(61.7-58.9)-1}{2.1\sqrt{\frac{1}{9}+\frac{1}{12}}}$	M1 m1		difference of means over sd correct form of sd
	= 1.94 1.94 > 1.6449 so reject H_0. There is sufficient evidence at the 5% level to support the trainer's claim	A1 A1√	7	CAO; AWRT ft on sample value and CV