A Level Statistics
AQA Past Exam Questions

TOPIC: The Normal Distribution

For the new specification, students can now use the Casio Claswiz calculator to find Normal probabilities. For old AQA questions this was not the case and more work was involved for the students. Therefore, some of the questions will be worth a lot more marks than will be on offer in an up to date Edexcel exam

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions ***on paper***
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Unless otherwise stated, statistical tests should be carried out at the 5% significance level.
- When a calculator is used, the answer should be given to three significant figures unless otherwise stated.

Information

- ***You may use the*** booklet 'Statistical Formulae and Tables'
- There are **13** questions in this question paper. The total mark for this paper is ***217***
- The marks for **each** question are shown in brackets – use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.
- Check your answers if you have time at the end.

AQA_JAN_2012_3

During June 2011, the volume, X litres, of unleaded petrol purchased per visit at a supermarket's filling station by private-car customers could be modelled by a normal distribution with a mean of 32 and a standard deviation of 10 .

(a) Determine:

(i) $P(X < 40)$;

(ii) $P(X > 25)$;

(iii) $P(25 < X < 40)$.

(7 marks)

(b) Given that during June 2011 unleaded petrol cost £1.34 per litre, calculate the probability that the unleaded petrol bill for a visit during June 2011 by a private-car customer exceeded £65 .

(3 marks)

(c) Give two reasons, in context, why the model $N(32, 10^2)$ is unlikely to be valid for a visit by any customer purchasing fuel at this filling station during June 2011.

(2 marks)

AQA_JAN_2013_2

The volume of Everwhite toothpaste in a pump-action dispenser may be modelled by a normal distribution with a mean of 106 ml and a standard deviation of 2.5 ml.

Determine the probability that the volume of Everwhite in a randomly selected dispenser is:

(a) less than 110 ml;

(3 marks)

(b) more than 100 ml;

(2 marks)

(c) between 104 ml and 108 ml;

(3 marks)

(d) not exactly 106 ml.

(1 mark)

AQA_JUNE_2012_5

A general store sells lawn fertiliser in 2.5 kg bags, 5 kg bags and 10 kg bags.

(a) The actual weight, W kilograms, of fertiliser in a 2.5 kg bag may be modelled by a normal random variable with mean 2.75 and standard deviation 0.15 .

Determine the probability that the weight of fertiliser in a 2.5 kg bag is:

(i) less than 2.8 kg;

(ii) more than 2.5 kg.

(5 marks)

(b) The actual weight, X kilograms, of fertiliser in a 5 kg bag may be modelled by a normal random variable with mean 5.25 and standard deviation 0.20 .

(i) Show that $P(5.1 < X < 5.3) = 0.372$, correct to three decimal places.

(2 marks)

(ii) A random sample of four 5 kg bags is selected. Calculate the probability that none of the four bags contains between 5.1 kg and 5.3 kg of fertiliser.

(2 marks)

(c) The actual weight, Y kilograms, of fertiliser in a 10 kg bag may be modelled by a normal random variable with mean 10.75 and standard deviation 0.50 .

A random sample of six 10 kg bags is selected. Calculate the probability that the mean weight of fertiliser in the six bags is less than 10.5 kg.

(4 marks)

AQA_JUNE_2013_2
The weight, X grams, of the contents of a tin of baked beans can be modelled by a normal random variable with a mean of 421 and a standard deviation of 2.5 .
(a) Find:
(i) $P(X = 421)$;
(ii) $P(X < 425)$;
(iii) $P(418 < X < 424)$.

(6 marks)

(b) Determine the value of x such that $P(X < x) = 0.98$.

(3 marks)

(c) The weight, Y grams, of the contents of a tin of ravioli can be modelled by a normal random variable with a mean of m and a standard deviation of 3.0.
Find the value of m such that $P(Y < 410) = 0.01$

(4 marks)

AQA_JAN_2007_6
When Monica walks to work from home, she uses either route A or route B.
(a) Her journey time, X minutes, by route A may be assumed to be normally distributed with a mean of 37 and a standard deviation of 8.
Determine:
(i) $P(X < 45)$

(3 marks)

(ii) $P(30 < X < 45)$

(3 marks)

(b) Her journey time, Y minutes, by route B may be assumed to be normally distributed with a mean of 40 and a standard deviation of σ
Given that $P(Y > 45) = 0.12$, find the value of σ

(4 marks)

(c) If Monica leaves home at 8.15 am to walk to work hoping to arrive by 9.00 am, state, with a reason, which route she should take.

(2 marks)

(d) When Monica travels to work from home by car, her journey time, W minutes, has a mean of 18 and a standard deviation of 12.
Estimate the probability that, for a random sample of 36 journeys to work from home by car, Monica’s mean time is more than 20 minutes.

(4 marks)

(e) Indicate where, if anywhere, in this question you needed to make use of the Central Limit Theorem.

(1 mark)

AQA_JAN_2010_1
Draught excluder for doors and windows is sold in rolls of nominal length 10 metres. The actual length, X metres, of draught excluder on a roll may be modelled by a normal distribution with mean 10.2 and standard deviation 0.15 .
(a) Determine:
(i) $P(X < 10.5)$

(3 marks)

(ii) $P(10.0 < X < 10.5)$

(3 marks)

(b) A customer randomly selects six 10-metre rolls of the draught excluder.
Calculate the probability that all six rolls selected contain more than 10 metres of draught excluder.

(3 marks)

AQA_JAN_2008_1
In large-scale tree-felling operations, a machine cuts down trees, strips off the branches and then cuts the trunks into logs of length X metres for transporting to a sawmill.
It may be assumed that values of X are normally distributed with mean m and standard deviation 0.16 , where m can be set to a specific value.
(a) Given that μ is set to 3.3, determine:
(i) P(X < 3.5;

(3 marks)

(ii) P(X > 3.0)

(3 marks)

(iii) P(3.0 < X < 3.5)

(2 marks)

(b) The sawmill now requires a batch of logs such that there is a probability of 0.025 that any given log will have a length less than 3.1 metres.
Determine, to two decimal places, the new value of μ

(4 marks)

AQA_JAN_2009_3
UPVC facia board is supplied in lengths labelled as 5 metres. The actual length, X metres, of a board may be modelled by a normal distribution with a mean of 5.08 and a standard deviation of 0.05 .
(a) Determine:
(i) P(X < 5)

(3 marks)

(ii) P(5 < X < 3.1)

(2 marks)

(b) Determine the probability that the mean length of a random sample of 4 boards:
(i) exceeds 5.05 metres;

(4 marks)

(ii) is exactly 5 metres.

(1 mark)

(c) Assuming that the value of the standard deviation remains unchanged, determine the mean length necessary to ensure that only 1 per cent of boards have lengths less than 5 metres.

(4 marks)

AQA_JAN_2011_6
The volume of shampoo, V millilitres, delivered by a machine into bottles may be modelled by a normal random variable with mean m and standard deviation s.
(a) Given that μ = 412 and σ = 8 , determine:
(i) P(V < 400);

(3 marks)

(ii) P(V > 420);

(2 marks)

(iii) P(V = 410).

(1 mark)

(b) A new quality control specification requires that the values of μ and σ are changed so that

$$P(V < 400) = 0.05 \text{ and } P(V > 420) = 0.01$$

(i) Show, with the aid of a suitable sketch, or otherwise, that

$$400 - \mu = -1.6449\sigma \text{ and } 420 - \mu = 2.3263\sigma$$

(3 marks)

(ii) Hence calculate values for μ and σ

(3 marks)

AQA_JUNE_2007_7
(a) Electra is employed by E & G Ltd to install electricity meters in new houses on an estate. Her time, X minutes, to install a meter may be assumed to be normally distributed with a mean of 48 and a standard deviation of 20.
Determine:
(i) P(X < 60);

(2 marks)

(ii) P(30 < X < 60);

(3 marks)

(iii) the time, k minutes, such that P(X < k) = 0.9 .

(4 marks)

(b) Gazali is employed by E & G Ltd to install gas meters in the same new houses. His time, Y minutes, to install a meter has a mean of 37 and a standard deviation of 25.
(i) Explain why Y is unlikely to be normally distributed.

(2 marks)

(ii) State why Y, the mean of a random sample of 35 gas meter installations, is likely to be approximately normally distributed.

(1 mark)

(iii) Determine P(Y > 40).

(4 marks)

AQA_JUNE_2008_5
When a particular make of tennis ball is dropped from a vertical distance of 250 cm on to concrete, the height, X centimetres, to which it first bounces may be assumed to be normally distributed with a mean of 140 and a standard deviation of 2.5 .
(a) Determine:
(i) P(X < 145)

(3 marks)

(ii) P(138 < X < 142)

(4 marks)

(b) Determine, to one decimal place, the maximum height exceeded by 85% of first bounces.

(4 marks)

(c) Determine the probability that, for a random sample of 4 first bounces, the mean height is greater than 139 cm.

(4 marks)

AQA_JUNE_2009_3
The weight, X grams, of talcum powder in a tin may be modelled by a normal distribution with mean 253 and standard deviation σ
(a) Given that σ = 5 , determine:
(i) P(X < 250)

(3 marks)

(ii) P(245 < X < 250)

(2 marks)

(iii) P(X = 245)

(1 mark)

(b) Assuming that the value of the mean remains unchanged, determine the value of s necessary to ensure that 98% of tins contain more than 245 grams of talcum powder.

(4 marks)

AQA_JUNE_2010_3

Each day, Margot completes the crossword in her local morning newspaper. Her completion times, X minutes, can be modelled by a normal random variable with a mean of 65 and a standard deviation of 20 .

(a) Determine:

(i) $P(X < 90)$;

(ii) $P(X > 60)$.

(5 marks)

(b) Given that Margot's completion times are independent from day to day, determine the probability that, during a particular period of 6 days:

(i) she completes one of the six crosswords in exactly 60 minutes;

(1 mark)

(ii) she completes each crossword in less than 60 minutes;

(3 marks)

(iii) her mean completion time is less than 60 minutes.

(4 marks)

AQA_JUNE_2011_2

The diameter, D millimetres, of an American pool ball may be modelled by a normal random variable with mean 57.15 and standard deviation 0.04 .

(a) Determine:

(i) $P(D < 57.2)$;

(3 marks)

(ii) $P(57.1 < D < 57.2)$.

(2 marks)

(b) A box contains 16 of these pool balls. Given that the balls may be regarded as a random sample, determine the probability that:

(i) all 16 balls have diameters less than 57.2 mm;

(2 marks)

(ii) the mean diameter of the 16 balls is greater than 57.16 mm.

(4 marks)

AQA_JUNE_2015_2

The length of aluminium baking foil on a roll may be modelled by a normal distribution with mean 91 metres and standard deviation 0.8 metres.

(a) Determine the probability that the length of foil on a particular roll is:

(i) less than 90 metres;

(ii) not exactly 90 metres;

(iii) between 91 metres and 92.5 metres.

[6 marks]

(b) The length of cling film on a roll may also be modelled by a normal distribution but with mean 153 metres and standard deviation s metres. It is required that 1% of rolls of cling film should have a length less than 150 metres. Find the value of s that is needed to satisfy this requirement.

[4 marks]

AQA_JUNE_2014_2
(a) Tim rings the church bell in his village every Sunday morning. The time that he spends ringing the bell may be modelled by a normal distribution with mean 7.5 minutes and standard deviation 1.6 minutes.
Determine the probability that, on a particular Sunday morning, the time that Tim spends ringing the bell is:
(i) at most 10 minutes;
(ii) more than 6 minutes;
(iii) between 5 minutes and 10 minutes.

[6 marks]

(b) June rings the same church bell for weekday weddings. The time that she spends, in minutes, ringing the bell may be modelled by the distribution $N(\mu , 2.4^2)$. Given that 80 per cent of the times that she spends ringing the bell are less than 15 minutes, find the value of μ

[4 marks]

AQA_JUNE_2017_3
A common breed of turkey is the Large White.
(a) The weight, in kilograms, of a female Large White turkey, at 20 weeks old, can be modelled by a normal distribution with a mean of 8.25 kg and a standard deviation of 1.25 kg.
A 20-week-old female Large White turkey is selected at random.
Determine the probability that this turkey weighs:
(i) at most 9.00 kg;
(ii) more than 8.00 kg;
(iii) not exactly 8.25 kg;
(iv) between 8.00 kg and 10.00 kg.

[7 marks]

(b) The weight, in kilograms, of a male Large White turkey, at 24 weeks old, can be modelled by the distribution $N(\mu , \sigma^2)$.
Given that, for such turkeys, 2.5% weigh less than 10 kg and 2.5% weigh more than 20 kg, determine values for μ and σ .

[4 marks]

AQA_JUNE_2018_3
(a) The weight of cheese, X , in a large pack can be modelled by a normal random variable with a mean of 365.0 grams and a standard deviation of 10.0 grams.
Determine the probability that the weight of cheese in a randomly selected large pack is:
(i) less than 377.5 grams;
(ii) more than 367.5 grams;
(iii) between 365.0 grams and 367.5 grams.

[6 marks]

(b) The weight of cheese, Y , in an extra-large pack can be modelled by a normal random variable with mean 475.0 grams, unknown standard deviation σ grams and $P(Y < 450.0) = 0.05$.
(i) Find, to the nearest 0.1 gram, the value of σ .

[3 marks]

(ii) Determine the probability that in a sample of 12 randomly selected extra-large packs:
(A) every pack contains more than 450.0 grams of cheese;
(B) the mean weight of cheese per pack is more than 470.0 grams.

[5 marks]