A Level Statistics

AQA Past Exam Questions

SOLUTIONS

TOPIC: The Poisson Distribution

For the new specification, students can now use the Casio Claswiz calculator to find Poisson probabilities. For old AQA questions this was not the case and more work was involved for the students. Therefore, some of the questions will be worth a lot more marks than will be on offer in an up to date Edexcel exam

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have retrievable mathematical formulae stored in them.

Instructions

• Use **black** ink or ball-point pen.

• If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.

• **Fill in the boxes** at the top of this page with your name, centre number and candidate number.

• Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.

• Answer the questions ***on paper***

• You should show sufficient working to make your methods clear. Answers without working may not gain full credit.

• Unless otherwise stated, statistical tests should be carried out at the 5% significance level.

• When a calculator is used, the answer should be given to three significant figures unless otherwise stated.

Information

• ***You may use the*** booklet 'Statistical Formulae and Tables'

• There are **14** questions in this question paper. The total mark for this paper is ***124***

• The marks for **each** question are shown in brackets – use this as a guide as to how much time to spend on each question.

Advice

• Read each question carefully before you start to answer it.

• Try to answer every question.

• Check your answers if you have time at the end.

• If you change your mind about an answer, cross it out and put your new answer and any working underneath.

• Check your answers if you have time at the end.

AQA_JUNE_2015_7

(a)	$p = 1 - (\text{sum of Ps}) = 0.12$ p is probability that Angus has to do all the work alone	B1 E1		Anything conveying this concept
			2	
(b)	$E(X) = 0\times p + 1\times0.15 + 2\times0.2 + 3\times0.21 + 4\times0.18 + 5\times0.14 = 2.6$ $E(X^2) = 0^2\times p + 1^2\times0.15 + 2^2\times0.2 + 3^2\times0.21 + 4^2\times0.18 + 5^2\times0.14\ (= 9.22)$ $\text{Var}(X) = \text{'}9.22\text{'} - \text{'}2.6\text{'}^2\ (= 2.46)$ $\text{s.d} = \sqrt{2.46} = 1.57$	M1 A1 M1 m1 A1		Or B2 for answer alone Complete method incl $- E(X)^2$ Or similar totally correct working AG SC: $E(X^2) = 9.22$ with no explanation followed by $\text{Var}(X) = 9.22 - 2.6^2$, $\text{s.d} = \sqrt{2.46} = 1.57$ earns B2 SC: $\text{s.d.} = \sqrt{(9.22 - 2.6^2)}$ with no explanation for 9.22 earns B1
			5	

AQA_JAN_2012_1

1 (a)	$P(X < 2) = P(X \le 1)$. $P = 0.231(1)$	M1 A1	 2	Award for 0.267 or 0.199 from adjacent columns seen
(b)	Use of Po(14) $P(X \le 8) - P(X \le 7)$ $= 0.0621 - 0.0316 = 0.0305$ Calculator $\rightarrow$ 0.0304	B1 M1 A1	 3	 Must be 8 – 7 Or formula applied to relevant λ 0.0304 to 0.0305
(c)	Use of Po(12) $1 - P(X \le 15)$ $1 - 0.8444 = 0.1556$	B1 M1 A1	 3	 0.156
(d)	Tyres will often be sold in multiples. So not independent as required by Poisson **or** Garage has limited stock of tyres/time to change tyres Poisson is not limited **or** Rate of sales not likely to be constant through the day	E1 E1 E1 E1 E1	 2	NB. Not 'customers are not independent', or 'tyres & other product not independent' Must be clearly tied to restriction of context, not simply 'Poisson can be infinite, number of tyres cannot be' Must tie to context. Not simply 'mean must be constant'
	Total		**10**	

AQA_JAN_2013_5

5 (a)	Mean = $1 \times 0.03 + 2 \times 0.12$ + etc. = 3.51 $E(X^2) = 1 \times 0.03 + 4 \times 0.12 + \ldots$ $Var(X) = E(X^2) - E(X)^2$ =1.0299	M1 M1 m1 A1	**4**	 **Applied** in this case AWRT 1.03
(b)(i)	0.51	B1	**1**	
(ii)	Mode = 3 $P(X \geq 3) = 0.85$	M1 A1	**2**	
(ii)	Median = 4 $P(X < 4) = 0.49$	M1 A1	**2**	
(c)(i)	Poisson would have significant probability of greater than 5 which does not match the context.	E1	**1**	Or similar reasoning in context.
(ii)	Mean of B(5, 0.7) = 3.5 variance of B(5, 0.7) = 1.05 So (a) answers good match	B1 B1 E1	**3**	 Must have both B1
		Total	**13**	

AQA_JUNE_2013_1

1(a)(i)	$P(\leq 4) - P(\leq 3) = 0.8774 - 0.7360$	M1		Or by using formula or calculator Allow for use of adjacent columns, 0.8477 – 0.6919 = 0.1558 or 0.9041 – 0.7787 = 0.1254 or a hybrid eg 0.8774 – 0.6919
	= 0.141(4)	A1	2	AWFW 0.141-0.142 Unsupported answer scores B2
(ii)	1 – P(= 0) using Po(2.6)	M1		
	= 0.926 (0.9257)	A1	2	AWFW 0.925 to 0.926 Unsupported answer scores B2
(b)	Using Po(13)	M1		Stated or sight of 0.675, **0.764**, 0.836
	$P(\leq 15) = 0.764$ (0.7636)	A1	2	AWFW 0.763 to 0.764 Unsupported answer scores B2
(c)	Using Po(6)	B1		Stated or sight of 0.744, **0.847**, 0.919 or 0.356, **0.153**, 0.094
	$1 - P(\leq 8)$	M1		
	= 0.153 (0.1528)	A1	3	AWFW 0.152 to 0.153 Unsupported answer scores B3 SC Use of 1 – 0.9917 (= 0.0083) scores 1
	Total		**9**	

AQA_JUNE_2014_2

2(a)(i)	$P(<4) = P(\leq 3) = 0.558(4)$	B1	1	AWFW 0.558 to 0.559
(ii)	Using Po(13) Use of $P(\leq 19) = 0.9573$ for top value subtract $P(\leq 10) = 0.2517$ for bottom value giving 0.7056 SC Stating that $P(\leq 19) - P(\leq 10)$ is required but using wrong value of λ earns a single M1	B1 M1 M1 A1	4	Stated or use of any of 0.1658, 0.2517,0.9573, 0.9750 Indep of previous M1 AWFW 0.705 to 0.706
(b)(i)	P(at least 1) = 1 – P(0) = 1 – 0.0183 = 0.9817 (or 0.982)	M1 A1	2	Attempt to apply in this case CAO
(ii)	$0.9817^2 \times 0.0183$ $\times 3$ = 0.0529	M1 m1 A1	3	Use of their (b)(i) AWFW0.052 to 0.053
(c)	Mean = 100 Standard deviation = $\sqrt{100} = 10$	B1 B1	2	CAO CAO
(d)	Because we can no longer assume independence.	E1	1	OE

AQA_JUNE_2017_2

(a)(i)	Using Po(3.2), $P(D \leq 3) - P(D \leq 2)$ $= 0.6025 - 0.3799$ or $e^{-3.2} \times 3.2^3/3!$ $= 0.2226$ (= 0.223 to 3 s.f.)	M1 A1	2	PI by correct answer AWFW 0.222 to 0.223 No working but correct scores B2
(ii)	Using Po(3.8), $P(T \leq 4) - P(T \leq 1)$ $= 0.6678 - 0.1074$ $= 0.5604$	M1 B1 A1	3	Stated or implied by correct answer For either of these to at least 3sf AWFW 0.560 to 0.561 No working but correct scores B3
(iii)	$P(D \leq 4) = 0.7806$ $P(T \leq 6) = 0.9091$ Product = 0.7096 = 0.710 to 3sf	B1 B1 B1	3	To at least 3sf To at least 3sf AWRT 0.710. Accept 0.71. No working but correct scores B3
(b)	Using Po(7) Require $P(\text{Total} \leq n) \geq 0.99$ Or use of inverse Poisson for 0.99 $P(\text{Total} \leq 13) = 0.9872$, $P(\text{Total} \leq 14) = 0.9943$ So 14 – 10 = 4 extra rooms	B1 M1 B1 A1dep	4	Stated or relevant probability seen PI by correct answer Either seen to at least 3sf Dep on previous B1. CAO No working but just 4 or 14 – 10 = 4 scores 0 out of 4
	Total		12	

AQA_JAN_2007_1

1(a)(i)	P(4 or fewer) = 0.4405	B1		0.4405 (0.440 ~ 0.441)
(a)(ii)	P(4) = P(≤4) – P(≤3) = 0.4405 – 0.2650 = 0.1755	M1 A1		P(4) = P(≤4) – P(≤3) or correct use of formula 0.1755 (0.175 ~ 0.176)
(a)(iii)	P(≥4) = 1 – P(≤3) = 1 – 0.265 = 0.735	M1 A1	 5	P(≥4) = 1 – P(≤3) or correct use of formula 0.735 (0.734 ~ 0.736)
(b)	Poisson mean 15 P(>12) = 1 − P(≤12) = 1 − 0.2676 = 0.732	B1 M1 A1	 3	use of Poisson 3 × 5 P(>12) = 1 – P(≤12) 0.732 (0.732 ~ 0.733)
	Total		**8**	

AQA_JAN_2010_5

5(a)	0.8335	B1	1	0.834 (0.833 ~ 0.834)
(b)(i)	0.0273	B1	1	0.0273 (0.027~0.0274)
(ii)	0.3027 - 0.1257 = 0.177	M1 A1	 2	P (2 or fewer) - P(1 or fewer) 0.177 (0.1765 ~ 0.1775)
(iii)	$P_o(5)$ P (>6) = 1 - 0.7622 = 0.2378	B1 M1 A1	 3	$P_o(5)$ method - their mean 0.238 (0.237 ~ 0.238)

AQA_JAN_2011_2

2(a)(i)	P(2 or fewer) = 0.5184	B1	1	B1 0.518 (0.518 ~ 0.519)
(ii)	P(> 3) = 1 – P(3 or fewer) = 1 – 0.7360 = 0.264	M1 A1	 2	M1 method A1 0.264 (0.2635 ~ 0.2645)
(iii)	P(4) = P(4 or fewer) – P(3 or fewer) = 0.8774 – 0.7360 = 0.1414	M1 A1	 2	M1 method A1 0.1414 (0.141 ~ 0.142)
(b)	Poisson mean 13 P(15 or fewer) – P(9 or fewer) = 0.7636 – 0.1658 = 0.598	B1 M1 m1 A1	 4	B1 poisson mean 5×2.6 M1 method — generous m1 correct method A1 0.598 (0.597 ~ 0.6)
	Total		**9**	

AQA_JUNE_2007_1

1(a)(i)	P(3 or fewer)=0.779	B1		0.779 (0.778~0.779)
(a)(ii)	P(3)=P(3 or fewer)−P(2 or fewer) =0.7787−0.5679 =0.209	M1 m1 A1	 4	$P(3)=P(\le 3)-P(\le 2)$ completely correct method 0.209(0.208~0.21)
(b)	Poisson mean $5\times 2.4=12$ P(>10)=1−P(10 or fewer) =1−0.3472 =0.653	B1 M1 A1	 3	Poisson mean 5×2.4 P(>10)=1−P(10 or fewer) 0.653 (0.652~0.653)
(c)	No, customers are likely to join shortest queue i.e. not at random.	E1 E1	 2	No Reason – allow not independent – couple may shop together etc.
	Total		**9**	

AQA_JUNE_2008_4

4(a)(i)	0.5488	B1		0.5488 (0.5485 ~ 0.5495)
(ii)	$P(\ge 3) = 1 - P(\le 2)$ $= 1 - 0.9769$ $= 0.0231$	M1 A1	 3	 0.0231 (0.023 ~ 0.0232)

AQA_JUNE_2009_1

1(a)(i)	0.5488	B1		0.5485 ~ 0.549
(ii)	$P(X > 2) = 1 - P(X \le 2)$ $= 1 - 0.9769$ $= 0.0231$	M1 A1	 3	 0.023 ~ 0.0232
(b)(i)	Po(0.3)	B1		Attempted use of Po(0.3)
(ii)	$P(X \ge 1) = 1 - P(X = 0)$ $= 1 - 0.7408$ $= 0.259$	M1 A1		 0.259 ~ 0.26
(ii)	$P(X = 2) = 0.9964 - 0.9631$ $= 0.0333$	M1 A1	 5	Method – includes Po(0.3) 0.033 ~ 0.0334
	Total		**8**	

AQA_JUNE_2010_6

6(a)(i)	1 - 0.8946 = 0.105	M1 A1		M1 P(6 or more) = 1 - P(5 or fewer) A1 0.105 (0.105 ~ 0.106)
(ii)	0.0408	M1 A1	4	M1 Attempt to find P(0) A1 0.0408 (0.0407 ~ 0.041)
(b)(i)	0.2689	B1		B1 0.269 (0.2688 ~ 0.269)
(ii)	Poisson mean 3.2 + 3.8 = 7 P(<2) = P(1 or fewer) = 0.0073	M1 m1 A1	 4	M1 attempt to use Poisson mean 3.2 + 3.8 or equivalent m1 completely correct method A1 0.0073 (0.007 ~ 0.0073)
(iii)	In this week the total of the number who did not attend on Tuesday and the number who did not attend on Thursday was 1. As shown in part (b) this was an extremely unlikely occurrence prior to the change of policy. Hence it is likely that the change of policy has improved attendance.	E1 E1 E1	 3	E1 Policy effective E1 Attempt at reference to relevant probability E1 complete answer
(c)(i)	Poisson has no upper limit. Number of absentees cannot exceed size of squad (probably about 16)	E1		E1 no upper limit
(ii)	Same member may miss both sessions due to illness/holiday	E1	2	E1 reason
	Total		**13**	

AQA_JUNE_2011_4

4(a)(i)	P(0) = 0.301	B1		B1 0.301 (0.3 ~ 0.3015)
(ii)	P(1) = 0.6626 – 0.3012 = 0.361	M1 A1		M1 method A1 0.361 (0.36 ~ 0.362)
(iii)	P(5 or more) = 1 – 0.9923 = 0.0077	M1 A1	 5	M1 method A1 0.0077 (0.0077 ~ 0.0078)
(b)	$\bar{x} = 1.2$ $s^2 = 2.436$ ($\sigma^2 = 2.382$)	B1 B1	 2	B1 1.2 CAO B1 2.44 (2.43 ~ 2.44) or 2.38 (2.37 ~ 2.39)
(c)(i)	proportion 0s observed = 0.489 not similar to 0.301	M1 A1	 2	M1 attempt to compare observed proportion of 0s with probability calculated in (a)(i) A1 correct comparison
(ii)	Mean and variance of observed values not similar.	E1	1	E1 reason – even if based on incorrect answers to (b)
(iii)	Most credit cards lost in the evening may be reported in first hour / cards lost over the weekend likely to be reported on Monday etc.	E1 E1	 2	E1 reference to valid property of Poisson distribution E1 plausible context
(d)	No. Since distribution appears not to be Poisson no evidence to support view that reports arrive at random times.	E1 E1	 2	E1 No E1 not Poisson → not random or explanation of why not random
	Total		**14**	