A Level Statistics

AQA Past Exam Questions

SOLUTIONS

TOPIC: Confidence Intervals and the Central Limit Theorem

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions ***on paper***
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Unless otherwise stated, statistical tests should be carried out at the 5% significance level.
- When a calculator is used, the answer should be given to three significant figures unless otherwise stated.

Information

- ***You may use the*** booklet 'Statistical Formulae and Tables'
- There are **11** questions in this question paper. The total mark for this paper is ***99***
- The marks for **each** question are shown in brackets – use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.
- Check your answers if you have time at the end.

AQA_JUNE_2018_1

(a)	$\bar{x} = 19.14 \quad s_{n-1}^2 = 29.1(2125)$	B1		For both. PI Allow for $s_{n-1} = 5.396(411)$ or $s_n^2 = 27.9564$ or $s_n = 5.287381$ All answers truncated or rounded to 3sf.
	$19.14 \pm t_{24;0.005} \dfrac{5.396}{\sqrt{25}}$	M1		Correct form including use of *t* distribution and **either** use of $\sqrt{25}$ with s_{n-1} **or** use of $\sqrt{24}$ with s_n PI
	with *t* = 2.797	B1		Allow 2.8
		A1		Completely correct numerical expression
	= 19.14 ± 3.01875			
	giving limits (16.121, 22.159)	A1	5	Either form 19.1 ± AWRT(3.02) OR 16.05 ~ 16.2 and 22.1 ~ 22.2
Note: Use of z (=2.5758) gives (16.36,21.92) for first B1 only				
(b)	Because, with new sample values, the sample SD might increase to such an extent that $t \times \frac{SD}{\sqrt{n}}$ is larger, making the interval wider.	E1		Some consideration of SD/variance/variation.
		E1	2	Needs recognition that SD may increase **and** clear acknowledgement that it may more than counteract the effect of increasing n (and decreasing t).
	Total		**7**	

AQA_JUNE_2015_5

5 (a) (i)	0.10 or 10%	B1		oe
			1	
(ii)	P(≥ 1 exclude μ) = 1 – P(None exclude μ) = $1 - (0.9)^5$ = 1 – 0.59049 = 0.40951	M1 M1 A1		Any p^5 with 0<p<1 seen For $1 - (0.9)^5$ or $1 - (0.1)^5$ (= 0.99999) 0.4 ~ 0.41
	Alternative Using B(5, 0.1) table P(≥ 1) = 1 – 0.5905 **or** P(> 1) = 1 – 0.9185 (= 0.0815) = 0.4095	 (M1) (M1) (A1)		 Seen or implied Either expression seen 0.4 ~ 0.41
			3	
(b) (i)	$10.280 \pm t_{5;0.05}\dfrac{0.021}{\sqrt{6}}$ with $t = 2.015$ = 10.280 ± 0.017(275) giving limits (10.263,10.297)	M1 B1 M1 A1		Correct form including $\sqrt{6}$ and use of t distribution or z = 1.64 ~ 1.65 Condone 10.325 ± (…) 2.01 ~ 2.02 Completely correct expression evaluated. 10.28cao ± (0.017 ~ 0.0175) may be implied by correct limits not necessarily to 3dp. Both limits to 3dp. No isw here. Condone truncation to 10.262.
			4	
(ii)	**10.325** is **outside** (or **above**) the interval So new programme seems effective (or mean time decreased)	B1 B1dep		Needs their CI basically correct. Condone small slip but CI must be below 10.325. Need only refer to upper limit. Correct conclusion and reason. Dependent on previous B1 **Note.** These last 2 marks are available after using normal in part (i) which gives (10.266, 10.294)
			2	
		Total	**10**	

AQA_JAN_2013_2

2 (a)(i)	Use of 2.5758 99% CI is $6.75 \pm 2.5758 \times \frac{1.29}{\sqrt{1403}}$ $= 6.75 \pm 0.088(7)$ $= (6.66, 6.84)$	B1 M1 m1 A1	4	Accept 2.57~2.58 (even if called t) Use of $\frac{1.29}{\sqrt{1403}}$ (= 0.0344) Correct interval, allow any Z **Either** for 6.75 ± (0.088 ~ 0.089) **or** AWRT 6.66 and 6.84
(ii)	Comparing upper limit of CI for males with lower limit of CI for females. There is a difference between males and females.	M1 A1	2	Comparing limits of two CIs Requires implication of "difference" and clearly implied comparison of their upper limit with 6.87

AQA_JAN_2012_5

5(a)	$\bar{x} = 63.18$ $s = 8.097$ 95% confidence interval for mean $63.18 \pm 2.228 \times 8.097/\sqrt{11}$ ie 63.18 ± 5.44 57.74 ~ 68.62	B1 M1 m1 B1 B1√ A1	6	B1 63.18 (63.15 ~ 63.2) and 8.097 (8.09 ~ 8.1) M1 their sd/√11 m1 method for interval - allow incorrect t-value or arithmetic error only B1 10 df B1√ 2.228 their df A1 57.7 (57.7 ~ 57.8) and 68.6 (68.6 ~ 68.7) allow in ± form
(b)	Statement 1: A. Statement 2: D. The confidence interval is certain to contain the mean time taken by members of the sample Statement 3: C. There is no reason why this should be true since confidence interval is for mean not individual values. It could conceivably be true by chance.	B1 B1 E1 B1 E1	5	B1 A B1 D E1 explanation B1 C - allow D if accompanied by a reasonably good explanation E1 explanation
	Total		**11**	

AQA_JUNE_2013_3

3(a)(i)	$\bar{x} = 95.625$ $s = 9.1798$ Use of $t_7 = 2.365$ 95%CI: $95.625 \pm 2.365 \times \dfrac{9.1798}{\sqrt{8}}$ $= 95.625 \pm 7.67$ or 7.68 – (87.9, 103.3) or (88.0, 103.3)	B1 B1 B1 B1 M1 A1	 6	95 ~ 96 9.1 ~ 9.2 or 8.5 ~ 8.7 For 7df either here or in part (b)(i). (May be implied) For $t = (2.36 \sim 2.37)$ Their $\bar{x}, s, t$. Needs $\dfrac{s_{n-1}}{\sqrt{8}}$ or $\dfrac{s_n}{\sqrt{7}}$ Depends on rounding. Need not be seen cao and cwo Note: 88.0 results from using $t = 2.3646\ldots$from a calculator
(ii)	92 is within the interval So mean time has *not* changed	Bdep F1 Bdep F1	 2	Correct conclusion and reason for their CI.

AQA_JAN_2007_2

2(a)	$\bar{x} = 260.25 \quad s = 41.337$ 90% confidence interval for mean $260.25 \pm 1.895 \times \dfrac{41.337}{\sqrt{8}}$ 260.25 ± 27.70 $232.6 \sim 287.9$	B1 B1 B1 B1√ M1 m1 A1	 7	260.25 (260 ~ 260.3) 41.337 (41.3 ~ 41.4) 7 df 1.895 (1.89 ~1.9) use of $\dfrac{\text{their s.d.}}{\sqrt{8}}$ - generous method - allow incorrect t 232.6 (232.5 ~ 233) and 287.9 (287.5 ~ 288) or 260.25 (260 ~ 260.5) and 27.7 (27.65 ~ 27.75)
(b)	times were a random sample from a normal distribution	E1 E1	 2	random normal – allow independent
			9	

AQA_JUNE_2017_4

4(a)	Use of $t_4 = 2.132$ (for 90% CI)	B1		2.13 ~ 2.14 PI
	Use of $t_4 = 4.604$ (for 99% CI)	B1		4.6 ~ 4.61 PI
	$198.8 \pm t \times \frac{1.4}{\sqrt{5}}$	M1		Seen or implied for either interval. Allow M1 for *t*-value from df = 5 or 95% confidence or one-tailed. *(See note.)*
	90% CI: 198.8 ± 1.33 OR (197.47, 200.13)	A1		**Either** 198.8 ± AWRT 1.3 **Or** AWRT (197.5. 200.1)
	99% CI: 198.8 ± 2.88 OR (195.92, 201.68)	A1		**Either** 198.8 ± AWRT 2.9 **Or** AWRT (195.9, 201.7)
	Special Case If 0/5 award B1 for stated 90% CI (197 or 198, 200) and award B1 for stated 99% CI (196, 202)			
Note	*Other acceptable t-values for M1: 1.533, 2.776, 3.747, 1.476, 2.015, 2.571, 3.365, 4.032*			
			5	
(b)(i)	0.10	B1		OE
(ii)	0.90	B1		OE
			2	
(c)	$(1 - 0.9) \times (1 - 0.99)$	M1		Seen or implied
	$= 0.001$	A1		OE
			2	
		Total	**9**	

AQA_JAN_2010_5

5(a)(i)	$\bar{x} = 134.51 \quad s = 1.0181$ 95% confidence interval for mean $134.51 \pm 2.262 \times 1.0181/\sqrt{10}$ 134.51 ± 0.728 $133.78 \sim 135.24$	B1 M1 m1 B1 B1√ A1	 6	B1 134.51 (134.5 ~ 134.52) and 1.0181 (1.018 ~ 1.02) M1 use of their s.d/$\sqrt{10}$ m1 correct method for t B1 9df B1√ 2.262 their df A1 133.78 and 135.24 from correct working AG
(ii)	As all lengths start with 13, 3sf is in effect 1sf which is too few. **or** Width of confidence interval is 1.456 - if the limits had been rounded to 3sf the width would have apparently been 1 – a large % error.	E1	1	E1 reason
(b)	**Statement 1** C; Interval is based on the sample mean. There is no reason why 95% of individual lengths should lie in the interval. There is a very small possibility that it could occur by chance. **Statement 2** C; this would be true for an interval based on a known population mean and s.d. using z. It is extremely unlikely to be true of an interval based on estimates and t. **Statement 3** D; the interval is centred on x and so is certain to contain x.	B1 E1 B1 E1 B1 E1	 6	B1 C E1 explanation - allow both marks for a good explanation even if option D chosen. B1 C E1 explanation-allow both marks for a good explanation even if option D chosen. B1 D E1 explanation
	Total		**13**	
	TOTAL		**75**	

AQA_JAN_2011_2

2(a)	$\bar{x} = 22.45 \quad s = 2.034$ 95% confidence interval for mean $22.45 \pm 2.262 \times 2.034/\sqrt{10}$ $22.45 \pm 1.455 \quad (1.45 \text{ to } 1.46)$ $21.0 \sim 23.9$	B1 B1B1 M1m1 A1	 6	B1 22.45 (22.4 ~ 22.5) and 2.034 (2.03 ~ 2.04) B1 9df B1 2.262 M1 method for c.i — their s.d. and t-value m1 correct method for c.i. their t-value A1 21.0 (20.95 ~ 21.05) and 23.9 (23.85~23.95) allow in ± form
(b)	95% confidence interval for mean $18.27 \pm 1.96 \times 1.638/\sqrt{55}$ 18.27 ± 0.433 $17.9 \sim 18.7$	 B1 M1 A1	 3	B1 1.96 or 2.004 ~ 2.009 M1 method for c.i A1 17.9 (17.8 ~ 17.9) and 18.7 (18.65 ~ 18.75) allow in ± form
(c)	Evidence to support Olivia's claim for this rodent as lower limit of confidence interval for rodents on island is above upper limit of confidence interval on mainland. Only one island examined and no evidence for other species	E1 E1 E1	 3	E1 statement supported for this rodent E1 relevant comparison of confidence intervals E1 note of caution
	Total		**12**	

AQA_JUNE_2007_1

1(a)	$\bar{x} = 1023.3 \quad s = 525.19$	B1		$1023.3(1020 \sim 1025)$ and $525.2(525 \sim 525.5)$
	95% confidence interval for mean	B1		8 df
	$1023.3 \pm 2.306 \times \frac{525.19}{\sqrt{9}}$	B1√		2.306 – their df
		M1		use of their s.d. / $\sqrt{9}$
	i.e. 1023.3 ± 403.7	m1		method for interval
	$(620, 1427)$	A1	6	$620(619 \sim 620)$ and $1427(1426.5 \sim 1427.5)$ or 1430 or $1023.3(1020 \sim 1025)$ and $403.7(403 \sim 404)$
(b)	As 1250 lies within the confidence interval, there is no reason to doubt the firm's claim.	B1		accept claim
		B1√	2	1250 within interval
	Total		**8**	

AQA_JUNE_2010_1

1(a)	Approximate 99% confidence interval			
	SD = $\sqrt{24}$ used	B1		$\sqrt{24}$ used for s.d.
	$24 \pm Z\text{-value} \times \sqrt{24}$	M1		C1 method, recognisable Z, their s.d.
	Z = 2.5758	B1		Accept 2.58
	24 ± 12.6 or $11.4 \sim 36.6$	A1	4	$24 \pm (12.6 \sim 12.7)$ or $(11.3 \sim 11.4)$ to $(36.6 \sim 36.7)$
(b)	Since 17 lies within the interval	B1		17 lies within CI
	Editors claim incorrect/not justified No evidence of a significant increase in mean number of births	E1√	2	correct conclusion their CI
	Total		**6**	